SEQUENCE

It can be defined as a list of ordered elements, where specific trend exists between them.

GEOMETRIC PROGRESSION:

- sequence of the form $a, ar, ar^2, ar^3, \ldots ar^n$

$\frac{ar^n}{ar^{n-1}} = r$ → common ratio (ar^n: initial term)

Example:

$\{b_n\} = (-1)^n$, $n = 0, 1, 2, 3, \ldots$

$(-1)^0, (-1)^1, (-1)^2, (-1)^3 = 1, -1, 1, -1$

$r = \frac{-1}{1} = -1 = \frac{1}{-1} = -1$

$a = 1,\ r = -1 \therefore 1(-1)^n = (-1)^n = \{b_n\}$

SUMMATION

DOUBLE SUMMATION:

- this method is used when the elements for summation have two indices and we need to add all indices one by one

Example:

$$\sum_{i=1}^{4}\sum_{j=1}^{3} ij = \sum_{i=1}^{4} i+2i+3i = \sum_{i=1}^{4} 6i$$

$$= 6(1)+6(2)+6(3)+6(4)$$

$$= 60$$

COUNTING

INCLUSION-EXCLUSION PRINCIPLE:

- if we consider A_1 and A_2 to be two sets
- $|A_1|$ ways to select an element from A_1
- $|A_2|$ ways to select an element from A_2
- then the number of ways to select an element in both sets is: $|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$

Example:

Given: Total applicants = 350
Major in CS = 250
Major in BBA = 147
Major in both = 51

How many majored in neither?

applicants majoring in either CS or BBA = 250 + 147

applicants majoring in neither = 350 − (250 + 147) = 34

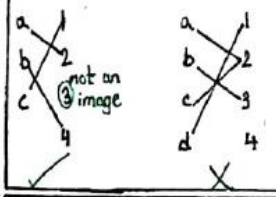
FUNCTIONS

A function from set A to set B is defined as $f: A \to B$ (or $f(a) = b$) where each element of set A will have a SINGLE image in B.

Types of functions with examples:

One-to-one (injective function):

$\forall a \forall b (f(a) = f(b) \to a = b)$



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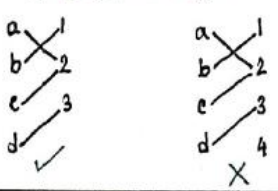
On-to (surjective function):

$\forall y \exists x (f(x) = y)$

co-domain = range



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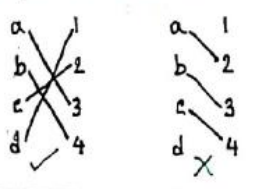
One-to-one correspondence (bijective)

$\forall y \exists x (f(x) = y)$

- the function is both one-to-one & onto



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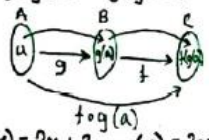
Composite functions:

$f: B \to C,\ g: A \to B$

$f \circ g(a) = f(g(a))$

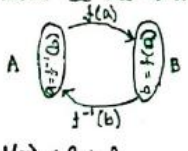


$f(x) = 2x + 3,\ g(x) = 3x + 2$

$f \circ g(x) = f(g(x)) = f(3x+2) = 2(3x+2)+3$

Inverse functions:

- must be one-to-one correspondence



$f(x) = 3x - 2$

$y = f(x) \Rightarrow x = f^{-1}(y) \Rightarrow y = 3x - 2$

$x = 3y - 2$ $\boxed{y = \frac{1}{3}(x+2) = f^{-1}(x)}$

Ceiling function:

- assigns to the next smallest integer that is greater than x

$x = 3.6$, $\text{ceil}(x) = \lceil x \rceil = \lceil 3.6 \rceil = \lceil 4 \rceil$

$\lceil 0.0001 \rceil = 1$

$\lceil 1 \rceil = 1$

Bytes of storage needed to store 100 bits of data: $\lceil \frac{100}{8} \rceil = \lceil 12.5 \rceil = 13$