COUNTING

Counting mainly encompasses fundamental counting rule, the Permutation rule and the combination rule.

The Rules of Sum and Product

The Rule of sum and the Rule of Product are used to decompose difficult counting Problems into simple problems.

The Rule of SUM

If a sequence of tasks $T_1, T_2, \ldots, T_m$ can be done in $\omega_1, \omega_2, \ldots, \omega_m$ ways respectively (the condition is that no task can be performed simultaneously), then the number of ways to do one of these tasks is $\omega_1+\omega_2+\ldots+\omega_m$. If we consider two tasks A and B which are disjoint (i.e $A \cap B = \emptyset$), then mathematically $|A \cup B| = |A| + |B|$

Example

Assume, there are three list for a computer project.

List 1: 23

List 2: 15

List 3: 19

No, project is on more than one list.

How many possible projects are there to choose from?

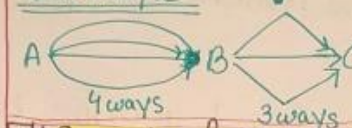
Ans:- $23+15+19$

$= 57$ ways

The RULE OF Product

If a sequence of tasks $T_1, T_2, \ldots, T_m$ can be done in $\omega_1, \omega_2, \ldots, \omega_m$ ways respectively and every task arrives after the occurrence of the previous task, then there are $\omega_1 \times \omega_2 \times \ldots \times \omega_m$ ways to ~~perm~~ perform the tasks. Mathematically, if a task B arrives after a task A, then, $|A \times B| = |A| \times |B|$

Example



So, there are $4 \times 3 = 12$ ways to reach, C from A.

Example of Rule of Sum and Product

A boy lives at X and wants to go to school at Z. From his home X he has to first reach Y and then Y to Z. He may go to X to Y by either 3 bus routes or 2 trains routes. From there, he can either choose 4 bus routes or 5 train routes to reach Z. How many ways are there to go from X to Z.

Soln:- From X to Y, he can go in $3+2=5$ ways
Thereafter, he can go Y to Z in $4+5=9$ ways } Rule of Sum

Hence, from X to Z, he can go in $5 \times 9 = 45$ ways → Rule of Product.

Inclusion-Exclusion Principle

Lets assume a task can be done in n_1 or n_2 ways. But some of n_1 ways to do the tasks are same as the some of the n_2 other ways to do the task. (in this situation) we also substract the number of ways to do the task that are among the n_1 and n_2 ways

Let's rephrase its using sets:- A_1 and A_2 are two sets.

$|A_1|$ ways to select an element from A_1. $|A_2|$ ways to select an element from A_2. So the no. of ways to select an element from A_1 or A_2 is.

$$|A_1 \cup A_2| = |A_1| + |A_2| - \underbrace{|A_1 \cap A_2|}$$

we substract the no. of ways to select an element common on both sets.

Example

Consider 350 applicants for a job, 220 majored in CS, 147 majored in Business, 51 majored in CS and Business both. So, How many of these applicants majored neither in CS nor in Business.

Soln: using the Inclusion-Exclusing Principle :-

$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2| = 220 + 147 - 51 = 316$

So, $350 - 316 = 34$ of the applicants major neither in Bus nor in CS.

Combinations:-

It is a selection of some given elements in which order does not matter. The no. of all combinations of n things, taken r at a time is

$$- \boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}$$

Example:

Find the no. of subsets of the set $\{1,2,3,4,5,6\}$ having 3 elements.

Soln:- ${}^6C_3 = \frac{6!}{3!(6-3)!} = 20$