SUMMATION OF THE TERMS OF A SEQUENCE

$$\sum_{j=m}^{n} a_j = a_m + a_{m+1} + \dots + a_n$$

The variable j is reffered to as the index of summation.

- m is the lower limit and
- n is the upper limit of the summation.

Example

1) Sum the first 7 terms of $\{n^2\}$ where $n = 1, 2, 3, 4, 5, 6, 7$

$$\sum_{j=1}^{7} a_j = \sum_{j=1}^{7} j^2 = 1+4+9+16+25+36+49 = 140$$

2) What is the value of $(-1)^j$

$$\sum_{K=4}^{8} a_j = \sum_{K=4}^{8} (-1)^j = 1+(-1)+1+(-1)+1 = 1$$

TYPES

There are two types of summation of a sequence. They are:-

→ Arithmetic series

→ Geometric series

Example

$$S = \sum_{j=1}^{5} (2+3j) = \sum_{j=1}^{5} 2 + \sum_{j=1}^{5} 3j =$$

$$= 2\sum_{j=1}^{5} 1 + 3\sum_{j=1}^{5} j =$$

$$= 2*5 + 3\sum_{j=1}^{5} j = 10 + \frac{3(5+1)\times 5}{2}$$

$$= 10 + 45$$

$$= 60$$

*Example 2

$$S = \sum_{j=3}^{5} (2+3j)$$

$$= \left[\sum_{j=1}^{5}(2+3j)\right] - \left[\sum_{j=1}^{2}(2+3j)\right] \leftarrow \text{Trick}$$

$$= \left[2\sum_{j=1}^{5} 1 + 3\sum_{j=1}^{5} j\right] - \left[2\sum_{j=1}^{2} 1 + 3\sum_{j=1}^{2} j\right]$$

$$= 55 - 13$$

$$= 42$$

ARITHMETIC SERIES

The sum of the arithmetic progression $a, a+d, a+2d, \dots, a+nd$ is called an arithmetic series.

Theorem: The sum of the terms of the arithmetic progression $a, a+d, a+2d, \dots, a+nd$ is,

$$\boxed{S = \sum_{j=1}^{n} (a+jd) = na + d\sum_{j=1}^{n} j = na + d\frac{n(n+1)}{2}}$$

SUMS

(Why) Proof:

$$S = \sum_{j=1}^{n} (a+jd) = \sum_{j=1}^{n} a + \sum_{j=1}^{n} jd = na + d\sum_{j=1}^{n} j$$

$$\sum_{j=1}^{n} j = 1+2+3+4+\dots+(n-2)+(n-1)+n$$

$$\underbrace{n+1 \quad n+1 \quad \cdots \quad n+1}_{n/2 \times (n+1)}$$

DOUBLE SUMMATION

$$S = \sum_{i=1}^{4} \sum_{j=1}^{2} (2i - j)$$

$$= \sum_{i=1}^{4} \left[\sum_{j=1}^{2} 2i - \sum_{j=1}^{2} j\right]$$

$$= \sum_{i=1}^{4} \left[2i\sum_{j=1}^{2} 1 - \sum_{j=1}^{2} j\right]$$

$$= \sum_{i=1}^{4} \left[2i*2 - 3\right]$$

$$= \sum_{i=1}^{4} 4i - \sum_{i=1}^{4} 3 =$$

$$= 4\sum_{i=1}^{4} i - 3\sum_{i=1}^{4} 1 = 4*10 - 3*4$$

$$= 28$$

GEOMETRIC SERIES

The sum of the terms of a geomatric progression $a, ar, ar^2, \dots, ar^k$ is called a geometric series.

Theorem: The sum of the term of a geometric progression $a, ar, ar^2, \dots ar^n$ is

$$S = \sum_{j=0}^{n} (ar^j) = a\sum_{j=0}^{n} r^j = a\left[\frac{r^{n+1}-1}{r-1}\right]$$

Proof: $S = \sum_{j=0}^{n} ar^j = a + ar + ar^2 + ar^3 + \dots + ar^n$

- multiply S by r

$$rS = r\sum_{j=0}^{n} ar^j = ar + ar^2 + ar^3 + \dots + ar^{n+1}$$

- Substract

$$rS - S = [ar + ar^2 + ar^3 + \dots + ar^{n+1}] - [a + ar + ar^2 + \dots + ar^n]$$

$$= ar^{n+1} - a$$

$$S = \frac{ar^{n+1} - a}{r-1} = a\left[\frac{r^{n+1}-1}{r-1}\right]$$

Example

$$S = \sum_{j=0}^{3} 2(5)^j$$

General formula:-

$$S = \sum_{j=0}^{n} (ar)^j$$

$$= a\sum_{j=0}^{n} r^j = a\left[\frac{r^{n+1}-1}{r-1}\right]$$

$$S = \sum_{j=0}^{3} 2(5)^j = 2*\frac{5^4-1}{5-1}$$

$$= 312$$

INFINITE GEOMETRIC SERIES

Infinite geometric series can be computed in a closed form for $x > 1$.

How?

$$\sum_{n=0}^{\infty} x^n = \lim_{K\to\infty} \sum_{n=0}^{K} x^n$$

$$= \lim_{K\to\infty} \frac{x^{K+1}-1}{x-1}$$

$$= \frac{-1}{x-1} = \frac{1}{1-x}$$

Thus, $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

An infinite geometric series is the sum of an infinite geometric series.

The sum S of an infinite geometric series with $-1 < r < 1$ given by the formula

$$S = \frac{a_1}{1-r}$$

An infinite series that has a sum is called Convergent series

S_n is called the partial sum of the series.

Example:- $\sum_{n=1}^{\infty} 10\left(\frac{1}{2}\right)^{n-1}$

$r = 1/2$ since, $|1/2| < 1$, the su[m] exists.

$$S = \frac{a_1}{1-r} = \frac{10}{1-1/2} = \frac{10}{(1/2)}$$

$$= 20$$