

Question 1: Given the equation $2x^2 + 5y^2 = 14$, prove that no integer solution exists for x and y .

Question 2: Prove that if $x^2 - 2a + 7$ is even, then x is odd. Assume that $a \in \mathbb{Z}$.

Question 3: Use contrapositive method to prove the statement: If $3n^2 + 4n + 1$ is even, then $3n + 1$ is even or $n + 1$ is even.

Question 4: Prove that no natural number n is both even and odd.

Question 5: Use mathematical induction process to prove the below mathematical statements $\forall n \geq 1$:

(a) $1 + 3 + 5 + \dots + (2n - 1) = n^2$

(b) $4^n - 1$ is a multiple of 3

(c) $2 + 4 + 6 + \dots + 2n = n(n + 1)$

(d) $-1 + 2 + 5 + 8 + \dots + (3n - 4) = (n/2)(3n - 5)$