RULES OF INFERENCE

Inference :

A conclusion based on the basis of evidence and reasoning.

Proofs in mathematics are valid arguments

- Argument : Sequence of statements end with conclusion
- Valid : Conclusion must follow from the truth of the preceeding sentences (statements)
  ↳ Called premises
- Falacies : Incorrect reasoning that generates or leads to invalid arguments.

Example

"If you have an updated RFID, you get the automated attendance"

"You have an updated RFID"

Therefore,

"You can get the automated attendance"

Here, Premises are both true.

Let p : You have an updated RFID
q : You can get the automated attendance

Then the argument has the form :

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore \ q \end{array}$$

here, ∴ denotes Therefore.

The argument form is valid
↓
when all premises are true, the conclusion is true.

Let p : " You have a current password "
q : " You can log onto the network "

Let's say, The argument is

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore \end{array}$$

Now. state all the propositions.

" If you have a current password, then you can log onto the n/w "
You have a current password

Therefore, 'You can log' onto the n/w.

When both $\begin{cases} p \to q \text{ and } p \text{ are true} \\ q \text{ must also be true} \end{cases}$

This is called valid form of Argument

↓ means that

Whenever all premises are true, the conclusion must also be true.

If p and $p \to q$ both are not true

Let's say p : You have access to the n/w

q : You can change your grade.

$p \to q$: If you have access to the n/w, you can change your grade "

This is false

p : True

So, p and $p \to q$ both are not true

Now,

If you have access to the n/w, then you can change your grade.

You have access to the n/w

∴ You can change your grade

False

argument

If we replace propositions by propositional variables, we obtain " Argument Form "

- As all the premises need to be true, a valid argument with premises $P_1, P_2, P_3 \ldots P_n$ and conclusion q.

$$(P_1 \wedge P_2 \wedge P_3 \wedge P_4 \ldots \wedge P_n) \longrightarrow p \text{ is a tautology}$$

- "If an Argument form is Valid"

Validity of an argument can be checked using Truth Table.

↓

An argument is valid if the conclusion is true whenever all the premises are true.

Important : ARGUMENT Vs. ARGUMENT FORM

Argument :

- is a sequence of propositional logic
- comprises of

Premises: All, but the final propositions are premises

Conclusion: Final proposition is called conclusion

- An argument is valid if the ~~truth~~ True of all premises implies that the conclusion is true.

↳ relates to tautology

Argument Form : When we replace propositions by propositional variables we obtain argument form.

- a sequence of compound propositions involving propositional variables
- Argument form is valid if no matter which particular propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true

Validity of an argument form

- An argument form, or in short an argument, is a sequence of statements.
- All statements but the last one are called premises or hypotheses.
- The final statement is the conclusion

An argument is valid if the conclusion is true whenever all the premises are true.

- Validity of an argument can be checked with truth table.

All the critical rows must correspond to the "True" value for conclusion.

↳ Where premises are true

Example :

Show $(p \vee q,\ p \to r,\ q \to r,\ \therefore r)$ is a valid argument.

p	q	r	$p \vee q$	$p \to r$	$q \to r$
T	T	T	T	T	$\boxed{T}$
T	T	F	T	F	F
T	F	T	T	T	$\boxed{T}$
T	F	F	T	F	T
F	T	T	T	T	$\boxed{T}$
F	T	F	T	T	F
F	F	T	F	T	T
F	F	F	F	T	T

critical rows

□ Show that the argument

$(p \to q, \therefore \sim p \to \sim q)$ is invalid

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \to \sim q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F (we have false; it fails)
F	F	T	T	T	T

So, the argument is invalid.

□ " If $\sqrt{2} > \frac{3}{2}$, then $(\sqrt{2})^2 > (\frac{3}{2})^2$. We know that $\sqrt{2} > \frac{3}{2}$. Consequently, $(\sqrt{2})^2 = 2 > (\frac{3}{2})^2 = \frac{9}{4}$ "

p : $\sqrt{2} > \frac{3}{2}$

q : $2 > (\frac{3}{2})^2$

here, Premises $p \to q$

p

and q is conclusion

It's a valid argument form

Modus Ponens

↳ Mode that affirms

Now, one of its premise is false.

So, we cannot conclude that the conclusion is true.

Also, conclusion itself is true.

More about valid argument

- An argument is valid if the truth of the premises logically guarantees the truth of the conclusion

You own either car or bike
You do-not own a car
Therefore, You own a bike

- Consider another argument:

FALSE { All microwave ovens are made of gold
All items made of gold are time-travel devices }
Therefore, all ovens are time-travel devices

FALSE

However, this is valid argument

Because

If the premises were true, their truth would logically guarantee the conclusions' truth.

A valid argument may still have a false conclusion

Sound argument:

A sound argument is one that is not only valid, but begins with premises that are actually true.

Rules of inference ...

It is below freezing now. Therefore, it is either freezing or raining now.

p : It is below freezing now

q : It is raining now

p	q	$p \vee q$	$p \rightarrow p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

$$\frac{p}{\therefore\ p \vee q}$$

This argument uses the additional rule.

- "It is below freezing and raining now". Therefore, "It is below freezing"

$$\frac{p \wedge q}{\therefore\ p}$$

This argument uses simplification rule.

For simplification, $(p \wedge q) \rightarrow p$ is the tautology

IMPORTANT :

Each valid logical inference rule corresponds to an implication that is a tautology.

"if p, then q" format

* "If you have a current password, then you can log on to the network"
* "You have a current password"

Therefore,

You log on to the network

This has the below form

$$\begin{array}{c} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Modus Ponens
↳ Mode that affirms

Comments: The argument can be valid even if one of the premises is false.
↳ For instance, You may or may not have a current password

* You can't log into the network
* If you have a current password, then you can log into the network

Therefore,

* You don't have a current password

We may think we may not be able to log in for many other reasons.

→ Cable problem
→ Computer problem
→ Other problems

Important :

When working with logic problems, it is important to take the statements literally.

Don't consider other problems that are not there or mentioned.

So, let's consider the argument again

You can't log into the network

If you have current password, then you can log into the network.

Therefore,

You don't have a current password

$$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore \neg p \end{array}$$

Modus Tollens

↳ Mode that denies

Here, one may not be able to log in because of numerous reasons. But

We only consider that you can not log in.

Hypothetical syllogism

Let's consider

"If I do not wake up early, then I miss the class"

"If I miss the class, then I do bad in Exam"

Therefore,

"If I do not wake up early, then I do bad in exam."

When the above is represented using propositional variables:

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore \; p \rightarrow r \end{array}$$

Conjunction rule of inference

Let p : You study hard

q : You do well in exam

$\therefore$ $p \wedge q$: You study hard and you do well in exam.

Example of application : Build Arguments

- "It is not sunny this afternoon and it is colder than yesterday"
- We will go swimming only if it is sunny
- If we do not go swimming, then we will take a canoe trip
- If we take a canoe trip, then we will be home by sunset

↓ lead to conclusion

We will be home by sunset

p: It is sunny this afternoon
q: It is colder than yesterday
r: We will go swimming
s: We will take a canoe trip
t: We will be home by sunset

Steps:

1.	$\neg p \wedge q$	Hypothesis
2.	$\neg p$	Simplification using 1
3.	$r \rightarrow p$	Hypothesis
4.	$\neg r$	Modus Tollens using 2 and 3
5.	$\neg r \rightarrow s$	Hypothesis
6.	s	Modus Ponens using 4 and 5
7.	$s \rightarrow t$	Hypothesis
8.	t	Modus Ponens using 6 and 7

Resolution Principle

Another approach to see if an argument is correct, we use resolution principle.

In this principle,

A variable or negation of a variable is called literal. say,

$p, \neg p$ etc.

Disjunction of literals is called a sum, and conjunction of literals is called product.

Clause : It is a disjunction of literals. so, it is a sum.

→ Clausal sentence :

it is either a literal, or a disjunction of literals. For instance

$p, \neg q, \neg p, q, \quad p \vee q$

are all clausal sentence.

More about clause

clause is the set of literals in the clausal sentences. Here

$\{p\}, \{\neg p\}, \underbrace{\{p, q\}}_{\equiv\, p \vee q}$ are clause

Empty clause: $\square$ (notation) ✓

Empty set $\{\ \}$ is also a clause

Resolution Principle continues ...

Resolution means

the quality of being determined or resolute.

So, we determine if an argument is correct, or the satisfiability.

Resolvant:

For instance,

two clauses

C_1 $\qquad$ C_2

literal L_1 $\qquad$ L_2 , where L_1 and L_2 are complementary to each-other.

So, the resolvant of C_1 and C_2 is obtained by

deleting L_1 and L_2 from C_1 and C_2

and

construct the disjunction of the remaining clauses.

For instance,

$$C_1 = P \vee Q \vee R \qquad C_1' = Q \vee R$$

$$C_2 = \neg P \vee \neg S \vee T \qquad C_2' = \neg S \vee T$$

Disjunction $\quad Q \vee R \vee \neg S \vee T$

☐ Resolution Principle continues ...

Given a set of clauses "S",

Resolution Principle statement

a (resolution) deduction of C from "S" is a finite sequence $C_1, C_2 \ldots C_K$ of clauses such that

each C_i either is a clause in S

or

a resolvent of clauses preceding C, and $C_K = C$

A deduction of $\square$ (empty clause) is called a refutation or a proof of "S"

→ We use resolution as refutation, means

- proof by contradiction using resolution.

- We start by assuming that opposite of the given will be true, and

we show that this leads to a contradiction, for the given premises

↳ Negation of the conclusion is inconsistent with the premise.

That's,

If you have an argument with premises

$P_1, P_2 \ldots \ldots \ldots P_r$ and C is the conclusion, to prove using resolution principle

... do the followings :

1. Put $P_1, P_2 \ldots P_r$ in clause form.
2. Add $\neg C$ in clause form. to 1

Now, If this sequence gives empty clause "$\square$", then the argument is valid.

Some examples ...

Convert $P \rightarrow (Q \wedge R)$ into clauses

First, eliminate "$\longrightarrow$". So, we obtain

$$P \rightarrow (Q \wedge R) \equiv \neg P \vee (Q \wedge R)$$
$$\equiv (\neg P \vee Q) \wedge (\neg P \vee R) \quad \text{distributive law}$$

Conjunction of disjunctions

Conjunctive of normal form.

So, we get two clauses

$(\neg P \vee Q)$

$(\neg P \vee R)$

As resolution principle can be used as a proof technique we use it prove "Modus pones"

$$\begin{array}{l} P \\ P \rightarrow Q \\ \hline \therefore Q \end{array}$$

Clauses

1.	P	Premise
2.	$\neg P \vee Q$	Premise
3.	$\neg Q$	Negation of conclusion
4.	Q	Resolvant of 1, 2
5.	$\square$	Resolvant 3, 4

- We run the resolution until we derive false
  or,
  we cannot apply the resolution rule anymore.

- Prove R.

Given
1. $P \vee Q$
2. $P \rightarrow R$ $\quad \neg P \vee R$
3. $Q \rightarrow R$ $\quad \neg Q \vee R$

So.

Step	Formula	Derivation
1.	$P \vee Q$	Given premise
2.	$\neg P \vee R$	Given premise
3.	$\neg Q \vee R$	Given Premise
4.	$\neg R$	Negation of conclusion
5.	$Q \vee R$	Resolution of 1 and 2
6.	$\neg P$	Resolution of 2 and 4
7.	$\neg Q$	Resolution of 3 and 4
8.	R	Resolution of 5 and 7
9.	$\square$	Resolution of 4 and 8

→ empty clause, which is false
→ desired contradiction

Proof strategies:

Prefer a resolution step involving an unit clause (clause with one literal)

Produce shorter clause — this is good as we are trying to produce zero-length clause,
→ contradiction

That's, unit preference rule says that

if you can involve a clause that has only one literal in it, that's usually a good idea.

It is good because you get back a shorter clause.

"Small is beautiful"
↳ short!

↓

Shorter the clause is, the closer it is to false.

Logic programming

- Developed by scientists from AI research area.
- Programs are written in language of logic

  ↓

  Prolog is the logic language

Prolog: It stands for "PROgramming Logic"

→ Prolog facts

→ Prolog rules

- Prolog facts define predicates by specifying elements that satisfy these predicates
- Prolg rules are used to define new predicates using those already defined by prolog facts.

Example

Let's say

John is father of Lilly
Kathy is mother of Lilly
Lily is mother of Bill
Ken is father of Karen

Who are grand parents of Bill ?
Who are grand parents of Karen ?

Let's define:

father (john, lily)
mother (Kathy, Lily)
mother (lily, Bill)
father (Ken, Karen)

Here, father, mother are predicates.

Statements like father (john, lily) is called an atomic formula (known as atom). These statements state true fact.

grandparent (X, Z) ≡ X is a grandparent of Z

Parent (X,Y) ≡ X is a parent of Y

Parent (X,Y) ≡ father (X, Y)

Parent (X,Y) ≡ Mother (X, Y)

Now, we write

Conditional statements

```
grandparent (X, Z) :-
                    parent (X,Y), parent (Y, Z)
Parent (X,Y) :- father (X,Y)
Parent (X,Y) :  mother (X, Y)
```

it can take any value; variables

This means,

For any X, Y, Z

if X is a parent of Y, and Y is a parent of Z, then X is a grand-parent of Z

Finally, the complete program

```
grandparent (X, Z) : parent (X,Y), Parent (Y, Z)
parent (X,Y) : father (X,Y)
parent (X,Y) : mother (X,Y)
father (john, lily)
mother (Kathy, lily)
mother (lily, bill)
father (Ken, Karen)
```

Prolog continues —

Ask something ...

?- grandparent (john, bill)

Prolog uses the reasoning and answer the query.
Here, Answer is YES.

?- grandparent (Q, Karen) Who are grand-parent of Karen?

Answer is NO

" Rule of Inference for Quantifiers "

Universal instantiation :

$$\text{Given} \quad \frac{\forall x\, P(x) \quad \text{Premise}}{\therefore\ P(c)}$$

Inference here is used to conclude that P(c) is true. Here, c is particular member of the domain.

" All students in this class study hard "

↓ we conclude

" x " studies hard.

Universal generalization

It states that

$\forall x\, P(x)$ is true,

given the premise $P(c)$ is true

$$\frac{P(c) \text{ for any arbitrary } c}{\therefore \forall x\, P(x)} \qquad c \in \text{domain.}$$

No control over c
No more assumption about c

Existential instantiation:

Concludes that there is an element "c" in the domain for which $P(x)$ is true if we know that $\exists x\, P(x)$ is true.

$$\frac{\exists x\, P(x)}{\therefore P(c) \text{ for some element } c}$$

Not arbitrary: It must be a $c \in$ domain that is making $P(c)$ true.

Existential Generalization:

This concludes that $\exists x\, P(x)$ is true when a particular element c with $P(c)$ true is known.

$$\frac{P(c) \text{ for some element}}{\therefore \exists x\, P(x)}$$

Example : Build Arguments with Quantifiers.

Given, " A student in this class has not read the book "

" Everyone in this class passed the first exam "

Therefore,

Someone who passed the first exam has not read the book.

$C(x)$: " x is in this class

$B(x)$: " x has read the book "

$P(x)$: " x passed the exam "

Step	Reason
1. $\exists x\,(C(x) \wedge \neg B(x))$	Premise
2. $C(a) \wedge \neg B(a)$	Existential instantiation from 1
3. $C(a)$	Simplification
4. $\forall x\,(C(x) \rightarrow P(x))$	Premise
5. $C(a) \rightarrow P(a)$	Universal instantiation 4
6. $P(a)$	Modus Ponens from 3 and 5
7. $\neg B(a)$	Simplification of 2
8. $P(a) \wedge \neg B(a)$	Conjunction from 6 and 7
9. $\exists x\,(P(x) \wedge \neg B(x))$	Existential generalization from 8