NOVEMBER 18, 2018 (BEGINNING OF THE CLASS) North South University

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Problem 1: Using Taylor's expansion, find the third-order polynomial for the below functions:

(a) $f(x) = \sqrt{x+1}$ about $x_0 = 0$.

(b) $f(x) = \sin(x)$ using $x_0 = \pi/2$.

Problem 2: For a given interval [0, 1], consider the following functions and draw their graphs (hand drawn is acceptable). Identify the fixed points for each:

(a) $f(x) = 1 - x$

(b) $f(x) = 0.5 \sin x$

(c) $f(x) = (2x - 1)^2$

Problem 3: For a rootfinding problem with $f(x) = x^2 - p = 0$, you are asked to use the iteration method to calculate the square root of a number p ($p > 0$). You are given two other functions, $A_1(x) = p + x - x^2$ and $A_2(x) = 1 + x - x^2/p$, where both $A_1(x), A_2$ can provide a fixed-point problem that is equivalent to the $f(x) = 0$. Suppose, we need to evaluate the square root of $p = 4$.

(a) Which function among the two will converge to $\sqrt{p} = 2$? Provide your explanation for each cases.

(b) Derive the fixed-point iteration function using Newton's method; consider p = 4.

Problem 4: Use fixed-point theorem studied in the class to show that-

(a) Unique fixed point exists between the interval [1/3, 1] for a function $f(x) = 2^{-x}$

(b) Calculate the number of iteration steps (n) needed to ensure an accuracy of approximation around 10^{-4}.

Problem 5: Find the Lagrange polynomial of degree-2 that interpolates $y = x^3$ at the three nodes $x_0 = 1$, $x_1 = 2$, $x_2 = 3$; represent the expression in the simplest possible form. Use MATLAB to compare the plot between $y = x^3$ and the interpolating polynomial you derive. Add the necessary plots and codes to your answer.