

Solution

Question 1: 6 Points

(a) Convert $(3.125)_{10}$ to binary representation.

		Remainder				
$3/2$	1	1	↑	$2 \times 0.125 = 0.250$	0	↓
				$2 \times 0.250 = 0.500$	0	
$1/2$	0	1		$2 \times 0.500 = 1.00$	1	

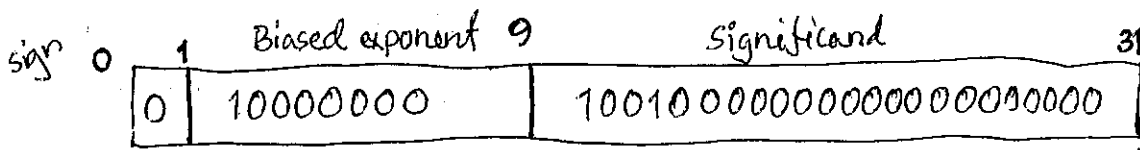
So, $(3.125)_{10} \equiv (11.001)_2$

11.001
 $\equiv 11.001 \times 2^0$

(b) Use IEEE 754 standard to represent the binary form of 3.125 to a 32-bit register.

So, Biased exponent = $127 + 1$ → coming from 2^1
 $= 128 \equiv (10000000)_2$

$\equiv 1.1001 \times 2^1$
 → implicit
 ↳ we generally don't put it in the register



Question 2: 6 Pts

Given $f(x) = x^{1/3}$ and $a = 4$, using Taylor's expansion find a polynomial of degree 2 for $f(x)$ at a .

$f(x) = x^{1/3}$	$f(4) = 4^{1/3}$
$f'(x) = \frac{1}{3} x^{1/3-1} = \frac{1}{3} x^{-2/3}$	$f'(4) = \frac{1}{3} 4^{-2/3}$
$f''(x) = \frac{1}{3} \left(-\frac{2}{3}\right) x^{-2/3-1} = -\frac{2}{9} x^{-5/3}$	$f''(4) = -\frac{2}{9} 4^{-5/3}$

Taylor's expansion of $f(x)$ at $x=a$

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \dots$$

For a polynomial of degree 2, we need upto x^2 term

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \dots$$

Bonus Question: 4 Points

Explain if the below sequence converges or diverges:

$$(a) \sum_{n=1}^{\infty} \frac{n^3}{n^5+3} \quad \frac{n^3}{n^5+3} < \left(\frac{n^3}{n^5} = \frac{1}{n^2} \right)$$

So, $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a converging sequence. Therefore, $\sum_{n=1}^{\infty} \frac{n^3}{n^5+3}$ is a converging sequence as well. as $\frac{1}{n^2}$ decreases as n increases.

(b) Given a function $f(x) = 4x^3 - 6x + 1 = 0$ identify if root for $f(x)$ exists within the intervals $[0, 1]$ and $[0, 2]$. What theorem guarantees the existence of a root within a given interval?

<p>For $[0, 1]$</p> $\begin{cases} f(0) = 1 \\ f(1) = -1 \end{cases}$ <p>opposite sign.</p>	<p>For $[0, 2]$</p> $\begin{aligned} f(0) &= 1 \\ f(2) &= 4 \times 8 - 6 \times 2 + 1 \\ &= 32 - 12 + 1 \\ &= 21 \end{aligned}$	<p>Root exists within the interval $[0, 1]$</p> <p>Intermediate value Theorem.</p>
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Question 3: 8 Points

Find the equation of the tangent line to $M(x) = (x \cos x)/(x+1)$ at the given point $x = \pi$

Given $M(x) = \frac{x \cos x}{x+1}$

$\Rightarrow \frac{d}{dx} M(x) = \frac{d}{dx} \frac{x \cos x}{x+1}$

$= \frac{-x \cos x \frac{d}{dx}(x+1) + (x+1) \frac{d}{dx} x \cos x}{(x+1)^2}$

$= \frac{(x+1) \left(\cos x \frac{d}{dx} x + x \frac{d}{dx} \cos x \right) - x \cos x}{(x+1)^2}$

$= \frac{(x+1) (\cos x - x \sin x) - x \cos x}{(x+1)^2}$

$= \frac{x \cos x - x^2 \sin x + \cos x - x \sin x - x \cos x}{(x+1)^2}$

$M'(x) = \frac{\cos x - x \sin x - x^2 \sin x}{(x+1)^2}$

sin +ve | all +ve
 $\frac{\pi}{2} \square \square \frac{\pi}{2}$
 tan +ve | cos +ve

So, $M(\pi) = \frac{\pi \cos \pi}{\pi+1} = -\frac{\pi}{\pi+1}$

$M'(\pi) = \frac{\cos \pi - \pi \sin \pi - \pi^2 \sin \pi}{(\pi+1)^2} = -\frac{1}{(\pi+1)^2}$

So, the point $(\pi, -\frac{\pi}{\pi+1})$

Eqⁿ for the tangent line is $y + \frac{\pi}{\pi+1} = m \cdot (x - \pi)$

$= -\frac{1}{(\pi+1)^2} (x - \pi)$