NAME AND ID: Solution

CSE 417, Quiz 1

MARKS OBTAINED:

Fall 2018, NSU

Question 1: 6 Points

(a) Convert $(3.125)_{10}$ to binary representation.

		Remainder
$3/2$	1	1
$1/2$	0	1

$2 \times 0.125 = 0.250 \quad 0$

$2 \times 0.250 = 0.500 \quad 0$

$2 \times 0.500 = 1.00 \quad 1$

so, $(3.125)_{10} \equiv (11.001)_2$

11.001

$\equiv 11.001 \times 2^0$

$\equiv 1.1001 \times 2^1$

implicit

we generally don't put it in the register

(b) Use IEEE 754 standard to represent the binary form of 3.125 to a 32-bit register.

So. Biased exponent $= 127 + 1$ coming from 2^1

$= 128 \equiv (10000000)_2$

sign 0	1 Biased exponent 9	Significand 31
0	10000000	10010000000000000000000

Question 2: 6 Pts

Given $f(x) = x^{1/3}$ and $a = 4$, using Taylor's expansion find a polynomial of degree 2 for $f(x)$ at a.

$f(x) = x^{1/3}$

$f'(x) = \frac{1}{3} x^{\frac{1}{3}-1} = \frac{1}{3} x^{-2/3}$

$f''(x) = \frac{1}{3}\left(-\frac{2}{3}\right) x^{-\frac{2}{3}-1} = -\frac{2}{9} x^{-5/3}$

$f(4) = 4^{1/3}$

$f'(4) = \frac{1}{3} 4^{-2/3}$

$f''(4) = -\frac{2}{9} 4^{-5/3}$

Taylor's expansion of $f(x)$ at $x=a$

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots$$

For a polynomial of degree 2, we need upto x^2 term

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \cdots \cdots$$

Bonus Question: 4 Points

Explain if the below sequence converges or diverges:

(a) $\sum_{n=1}^{\infty} \frac{n^3}{n^5+3}$ $\frac{n^3}{n^5+3} < \left(\frac{n^3}{n^5} = \frac{1}{n^2}\right)$

So, $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a converging sequence as $\frac{1}{n^2}$ decreases as n increases. Therefore, $\sum_{n=1}^{\infty} \frac{n^3}{n^5+3}$ is a converging sequence as well.

(b) Given a function $f(x) = 4x^3 - 6x + 1 = 0$ identify if root for $f(x)$ exists within the intervals [0 1] and [0 2]. What theorem guarantees the existence of a root within a given interval?

For [0 1]

$f(0) = 1$

$f(1) = -1$

opposite sign.

For [0 2]

$f(0) = 1$

$f(2) = 4\times 8 - 6\times 2 + 1$
$= 32 - 12 + 1$
$= 21$

Root exists within the interval [0 1]

Intermediate value Theorem.

Question 3: 8 Points

Find the equation of the tangent line to $M(x) = (x\cos x)/(x+1)$ at the given point $x = \pi$

Given $M(x) = \frac{x\cos x}{x+1}$

sin +ve | all +ve
tan +ve | cos +ve

$\Rightarrow \frac{d}{dx} M(x) = \frac{d}{dx} \frac{x\cos x}{x+1}$

$= \frac{-x\cos x \frac{d}{dx}(x+1) + (x+1)\frac{d}{dx} x\cos x}{(x+1)^2}$

$= \frac{(x+1)\left(\cos x \frac{d}{dx}x + x\frac{d}{dx}\cos x\right) - x\cos x}{(x+1)^2}$

$= \frac{(x+1)(\cos x - x\sin x) - x\cos x}{(x+1)^2}$

$= \frac{\cancel{x\cos x} - x^2\sin x + \cos x - x\sin x - \cancel{x\cos x}}{(x+1)^2}$

$M'(\pi) = \frac{\cos x - x\sin x - x^2\sin x}{(x+1)^2}$

So, $M(\pi) = \frac{\pi\cos\pi}{\pi+1}$
$= -\frac{\pi}{\pi+1}$

$M'(\pi) = \frac{\cos\pi - \pi\sin\pi - \pi^2\sin\pi}{(\pi+1)^2}$

$\Rightarrow M'(\pi) = -\frac{1}{(\pi+1)^2}$

So, the point $\left(\pi, -\frac{\pi}{\pi+1}\right)$

Eq^n for the tangent line is, $y + \frac{\pi}{\pi+1} = m.(x-\pi)$

$= \frac{-1}{(\pi+1)^2}(x-\pi)$