Question 1: 3 + 3 Points

(a) Find a fixed point for the below differential equation (Assume $p \in \mathbf{R}$):

$$\dot{x} = px - 4x^2 + x^3$$

(b) Identify the additional fixed points that exist for certain values of p. What are those values of p?

Question 2: 3 + 4 Points

(a) For a given interval $[-2\ \ 3]$, find a root of the below function using Bisection method:

$$f(x) = 3x^2 - 4x + 6$$

(b) Let's assume $f(x) = x^3 - \cos x$ and $m_0 = -1$. Apply the Newton's method to find m_2 (subscript 2 denotes the number of iteration step required).

Question 3: 3 + 5 Points

(a) Given $x = 1$ is a fixed point for a function $f(x) = x^3 - x^2 - 4x + 5$, does the fixed-point iteration scheme converge for x_0 close to 1? Justify your answer.

(b) For a given interval $[1 \ \ 2]$, your instructor MSK1 is asked by his supervisor to find a root of a function $f(x)$ within 6 minutes. Acceptable error bound allowed for the computation is 10^{-100}, and the computer MSK1 was using for the computation takes one second per iteration for the Bisection method. Comment (and justify) if MSK1 will be able to complete the computation within the given time.