NAME AND ID: Solution

CSE 417, Quiz 2

MARKS OBTAINED:

Fall 2018, NSU

Question 1: 3 + 3 Points

(a) Find a fixed point for the below differential equation (Assume $p \in R$):

$$\dot{x} = px - 4x^2 + x^3 = f(x)$$
$$= x(p - 4x + x^2)$$

Fixed point: $f(x) = x$.
Let's consider $x = 0$.
So, $f(x) = 0\,(p - 4\times 0 + 0^2)$
$= 0$

So, $x = 0$ is a fixed point

(b) Identify the additional fixed points that exist for certain values of p. What are those values of p?

Again, additional fixed points are from $(p - 4x + x^2) = 0$

$$\Rightarrow x = \frac{4 \pm \sqrt{16 - 4p}}{2}$$
$$\Rightarrow x = \frac{4 \pm 2\sqrt{4 - p}}{2}$$
$$\Rightarrow x = 2 \pm \sqrt{4 - p}$$

So, provided that $p \leq 4$, the other additional fixed points are $2 \pm \sqrt{4 - p}$

Question 2: 3 + 4 Points

(a) For a given interval $[-2 \;\; 3]$, find a root of the below function using Bisection method:

$$f(x) = 3x^2 - 4x + 6$$

$f(-2) = 3 \times (-2)^2 - 4(-2) + 6$
$\Rightarrow f(-2) = 3\times 4 + 8 + 6 = 26$

$f(3) = 3\times(3)^2 - 4\times 3 + 6$
$= 3\times 9 - 12 + 6$
$= 27 - 12 + 6 = 21$

As $f(x)$ has similar sign between the interval, root does not exist.

(b) Let's assume $f(x) = x^3 - \cos x$ and $m_0 = -1$. Apply the Newton's method to find m_2 (subscript 2 denotes the number of iteration step required).

$f'(x) = 3x^2 + \sin x$,

$m_1 = m_0 - \dfrac{f(m_0)}{f'(m_0)}$

$= -1 - \dfrac{(-1) - 0.54}{3 + 0.841}$

$= -1 + \dfrac{(1.54)}{3.841}$

$= -1 + 0.400$

$= -0.6$

$m_2 = m_1 - \dfrac{f(m_1)}{f'(m_1)} = -0.6 - \dfrac{1.04}{3\times(0.6)^2 + 0.565}$

$= -0.6 - \dfrac{1.04}{1.08 + 0.565}$

$= -0.6 - 0.63 = -1.232$

Question 3: 3 + 5 Points

(a) Given $x = 1$ is a fixed point for a function $f(x) = x^3 - x^2 - 4x + 5$, does the fixed-point iteration scheme converge for x_0 close to 1? Justify your answer.

Given, $f(x) = x^3 - x^2 - 4x + 5$. For fixed points

$f(x) = x$,

$\Rightarrow x^3 - x^2 - 4x + 5 = x$

$\Rightarrow x^3 - x^2 - 5x + 5 = 0$

$\Rightarrow x^2(x-1) - 5(x-1) = 0$

$\Rightarrow (x-1)(x^2-5) = 0$

$f'(x) = 3x^2 - 2x - 4$

For fixed point $x = 1$,

$|f'(1)| = |3 - 2 - 4| = 3 > 1$

So, fixed-point iteration does not converge.

(b) For a given interval [1 2], your instructor MSK1 is asked by his supervisor to find a root of a function $f(x)$ within 6 minutes. Acceptable error bound allowed for the computation is 10^{-100}, and the computer MSK1 was using for the computation takes one second per iteration for the Bisection method. Comment (and justify) if MSK1 will be able to complete the computation within the given time.

Total time given to MSK1 : $6 \times 60 = 360$ second

error bound of bisection says:

$$\frac{b-a}{2^n} < 10^{-100}$$

$$\Rightarrow \frac{2-1}{2^n} < 10^{-100}$$

$\Rightarrow \frac{1}{2^n} < 10^{-100}$

$\Rightarrow 2^n > 10^{100}$

$\Rightarrow n \log 2 > 100 \log 10$

$\Rightarrow n \times 0.301 > 100$

$\Rightarrow n > 333$

if one bisection iteration takes 1 second. Then,

333 bisection iteration takes 333 second,

< 360 second.

So, MSK1 can complete the computation