NAME AND ID: Solution & Necessary concepts. CSE 417, Quiz 3

MARKS OBTAINED: Fall 2018, NSU

Question 1: 6 + 8 Points

(a) What is the standard mathematical form of an Initial Value Problem (IVP)?

$$Y'(t) = \frac{d}{dt}Y(t) = f(t, y(t)), \quad a \leqslant t \leqslant b$$

$Y(a) = \alpha$ (alpha) → initial value

t : Independent variable

$y(t)$: Dependent variable

(b) Assume a differential equation $y'(t) = y^2 \sin(t)$ with values of $y(t)$ at $t = 0$ to be $y(0) = -3$. Use Euler method to approximate the value of $y(t)$ at $t = \pi/2$ by performing at least 2 iteration steps.

As we need at least two iterations, h could be $\pi/4$. See graphically

$$t \quad 0 \xleftrightarrow{\;h\;} \pi/4 \xleftrightarrow{\;h\;} \pi/4 + \pi/4 = \pi/2$$

So, $Y(0) = -3$ &

$h = \pi/4$

Here, $f(t, y(t)) = y^2 \sin(t)$

From Euler Method:

iteration 1: $\underbrace{Y_1}_{\text{at } \pi/4} = \underbrace{Y_0}_{=-3} + h f(\underbrace{t_0}_{=0}, \underbrace{Y_0}_{=-3})$

$$= -3 + \pi/4 \, (-3)^2 \sin(0) = -3 + 0 = -3$$

iteration 2:

$$\underbrace{Y_2}_{\text{at } \pi/2} = -3 + \pi/4 \, (-3)^2 \sin(\pi/4) = -3 + 4.9982$$

$$\simeq 1.9982 \quad \underline{\text{Ans}}$$

Here, $Y_2 \equiv Y(t)$ after two iterations, → takes us to $\pi/4 + \pi/4 = \pi/2$

Question 1: 8 Points

Given a function $f(x, y) = |y|$, check if it satisfies Lipschitz condition. If it satisfies, find out the Lipschitz constant (L). *Hints: You might find the formula of Triangle Inequality* $|a+b| \leq |a|+|b|$ *useful for this problem.*

According to Lipschitz condition

A fn^c $f(x,y)$ satisfies a Lipschitz condition in y on set $D \subset \mathbb{R}^2$ if there exists a constant $L>0$ such that $|f(x,y_1) - f(x,y_2)| \leq L|y_1 - y_2|$

Now, given $f(x,y) = |y|$

so, $f(x,y_1) = |y_1|$
$f(x,y_2) = |y_2|$

$\Rightarrow f(x,y_1) - f(x,y_2) = |y_1| - |y_2|$

$\Rightarrow |f(x,y_1) - f(x,y_2)| = \underline{||y_1| - |y_2||}$

we need to replace it with $|y_1 - y_2|$, and also, we need to obtain $\leq$

we can write:

$y_1 = y_1 - y_2 + y_2$

$\Rightarrow |y_1| = |\underbrace{y_2}_{a} + \underbrace{y_1 - y_2}_{b}|$

$\Rightarrow |y_1| \leq |y_2| + |y_1 - y_2|$ applying the Triangle Inequality

$\Rightarrow |y_1| - |y_2| \leq |y_1 - y_2| \quad \cdots \text{①}$

$y_2 = y_1 - y_1 + y_2$

$\Rightarrow y_2 = y_1 + y_2 - y_1$

$\Rightarrow |y_2| = |y_1 + y_2 - y_1|$
$\leq |y_1| + |y_2 - y_1|$
$\leq |y_1| + |y_1 - y_2|$

$\Rightarrow -|y_1 - y_2| \leq |y_1| - |y_2| \quad \cdots \text{②}$

From ① and ②, we can write

$-|y_1 - y_2| \leq |y_1| - |y_2| \leq |y_1 - y_2|$

Definition of

$\Rightarrow ||y_1| - |y_2|| \leq |y_1 - y_2|$

Lipschitz constant.

so, $|f(x,y_1) - f(x,y_2)| \leq |y_1 - y_2| \equiv 1 \cdot |y_1 - y_2|$

That is, $f(x,y) = |y|$ satisfies Lipschitz condition.