Midterm 1: CSE 257/417 Numerical Methods

North South University

September 1, 2018

Name: _____

Student ID: _____

Deduction due to misconduct:

Total Marks:

Instructions:

1. It is a close notes, close books exam.
2. You have **90 minutes** to complete the examination.
3. You may use a calculator.
4. Please sign the below Honor Code statement.

In recognition of and in the spirit of the North South University code of conduct, I certify that I will neither give nor receive unpermitted aid on this examination. Signature: _____

1 Question 1 (20 points)

Given the values $f(0) = 0$, $f(1) = 1$, $f(2) = 1$, $f(3) = 1$, find a polynomial that interpolates the points $(1, 2, 3, 4)$ by using Newton's divided difference formula. Make sure that you show the Newton table for divided difference.

2 Question 2 (20 points)

For $p > 0$, $p^{1/3}$ is the unique root of $x^3 - p = 0$. Now, answer the followings:

(a) Define the Newton's iteration to approximate the value of x

(b) For $p = 2$ and $x_0 = 1$, demonstrate two iterations to calculate x_1 and x_2 using Newton's iteration.

3 Question 3 (20 points)

(a) Determine if the function $4x^3 - 6x + 1 = 0$ has a real root in the interval [0 1]. If the root exists, implement 1 step of the Bisection method for the given interval [0 1].

(b) Reformulate the problem into a fixed point problem, and perform 1 step using the fixed point iteration method.

4 Question 4 (10 Points)

Find the limit of the below sequence:

$$\left(\sqrt{n^2+1}\right)/(2n+1) \text{ as } n \to \infty \tag{1}$$

5 Question 5 (20 points)

(a) Provide the basic representation of an Initial Value Problem (IVP). Why is it called an IVP?

(b) Suppose, you are asked to determine if an initial value problem (IVP) is well-posed or ill-posed, what would be your criteria to differentiate between the well-posed and ill-posed problem?

(c) For a given equation $\frac{dy}{dt} = f(t, y) = 2y^2 - (2t/y)$, and $D = \{(t, y) : -3 \leq t \leq 3, \ \ 1 \leq y \leq 5\}$

6 Question 6 (20 points)

(a) For an initial interval (a, b), obtain an equation for the number of iterations, n, needed by the Bisection method to converge within a given absolute error tolerance of ϵ. *Hint*: Use absolute error $|p_n - p| \leq (b - a)/2^n$, where p, P_n are the original and approximated root respectively.

(b) Using the formula you derive, calculate the number of steps n for $a = 0,\ b = 1,$ and $\epsilon = 0.5$.