Homework 1

Q1. Prove that $1+x+x^2+\cdots\cdots+x^{n-1}=\dfrac{1-x^n}{1-x}$, $\forall n\geq 1$ and $x\neq 1$.

Q2. Calculate $2+6x+12x^2+\cdots\cdots+n(n-1)x^{n-2}$

(hint: Take derivative of the sum $1+x+\cdots+x^{n-1}$ in the desired form)

Q3. Find out the sum of the geometric sequence: $\displaystyle\sum_{k=1}^{\infty}\frac{e^k}{3^{k-1}}$

Q4. State the Fubini's Theorem. Calculate

$$\iint_R \left(1+(x-1)^2+4y^2\right)dA, \text{ where } R=[0,4]\times[0,4]$$

Q5. Evaluate the two-dimensional integrals stated as below:

ⓐ $f(x,y)=\max(x,y)$ over a region enclosed by as $\{(x,y)\mid 0\leq x\leq 2 \text{ and } 0\leq y\leq 2\}$

ⓑ $\displaystyle\int_0^1\int_0^x \sqrt{1-x^2}\,dy\,dx$

Q5. Consider the double integral in cylindrical coordinates and find the volume under $z=\sqrt{4-r^2}$ above the circle part in the first quadrant, where circle equation is

$$x^2+y^2=9$$

Q6. Integration by parts:

ⓐ $\int \tan^{-1}x\,dx$

ⓑ $\int x^2e^x\,dx$

ⓒ $\int_0^{\infty} \lambda x e^{-\lambda x}\,dx$

ⓓ $\displaystyle\int_{-\infty}^{\infty} \frac{\lambda x}{2} e^{-\lambda|x|}\,dx$

counting

Q1. Prove that for all positive integers ($\mathbb{Z}^+$) n and K, where $n \geq K$, the below expression holds

Q2. Suppose, we have 10 students. We form three teams where Team 1 has 2 students

Team 2 has 4 "

Team 3 has 4 students

So, how many ways are there ~~two~~ to split these ten students into three teams as per the above requirement.

Q3. Suppose that ~~you~~ can make plain cake and ~~cakes with~~ five other flavors to be blended. Consider that you want to blend the flavors with the plain cake. How many different types of flavors can you come up with for your bakery?

Q4. Consider that we have 12 red dice and 6 white dice. Answer the followings:

a) How many ways are possible to choose 4 dices, where at least three (3) dices will be red.

b) How many choices of 4 dices are possible with at least one red dice in every choice is made.