CSE 422

Modeling And Simulation

Class Notes

Lecture : 2, 3 (Section : 1)

Submitted by:

1. Alamin Sheikh Naim - 2013556642

   Lecture : 2

2. Md. Jayeed Hasan Hriday - 2013009642

   Lecture : 3

Submitted to:

DR. MD SHAHRIAR KARIM, Assistant Professor, department of ECE, North South University.

Modeling And Simulation

Lecture: 2

Sequence and Series:

In mathematics, a geometric series is the sum of an infinite number of terms that have a constant ratio between successive terms. The formula of series is:

$a + ar + ar^2 + ar^3 + \dots$

Where, $a =$ start term

$r =$ common ratio

It is geometric because it maintain a certain ratio. Example:

$$\frac{ar}{a} = r = \frac{ar^2}{ar} \quad ; \text{ assuming, } a = 1$$

$1 + r + r^2 + r^3 + \dots + r^{n-1} + r^n$

$$= \sum_{k=1}^{n+1} = \sum_{i=0}^{n} r^2 = \frac{r^{n+1} - 1}{r - 1}, \quad r \neq 1$$

$k - 1 = i$

$i = (k=1) - 1$

$= 0.$

$n \to \infty$; Special case of geometric sequence

$|r| < 1$

$$= \frac{r^{n+1} - 1}{r - 1}$$

; Here n is approaching to infinity

$$= \frac{-1}{r - 1}$$

$$= \frac{1}{1 - r}$$

// $0.5^1 = 0.5$

$0.5^2 = 0.25$

$0.5^3 = 0.125$

Proof/calculate:

$$\sum_{k=2}^{\infty} \frac{1}{2^k} = ?$$

Solution: $\frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \dots + \frac{1}{2^n}$

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

$$= \frac{1}{4} + \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{8} + \dots$$

$$= \frac{1}{4}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right)$$

$$= \frac{1}{4}\left(1 + \underbrace{\left(\frac{1}{2}\right)}_{r} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots\right)$$

$$= \frac{1}{4}\cdot\frac{1}{1-r}$$

$$= \frac{1}{4} \times \frac{1}{1-\frac{1}{2}}$$

$$= \frac{1}{4} \times \frac{2}{1}$$

$$= \frac{1}{2}. \text{ (Ans.)}$$

Proof that: $\sum_{k=1}^{\infty} k r^{k-1} = 1 + 2r + 3r^2 + \dots = \frac{1}{(1-r)^2}$

Proof:

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$

$$\frac{d}{dr}\sum^{\infty} r^k = \sum_{k=0}^{\infty} \frac{d}{dr} r^k = \sum_{k=0}^{\infty} k r^{k-1} = \sum_{k=1}^{\infty} k r^{k-1}$$

Derivative with respect to r.

R.H.S – Derivative with respect to r.

$$\therefore \frac{d}{dr}\left(\frac{1}{1-r}\right) = \frac{d}{dr}(1-r)^{-1} + (1-r)^{-1-1}\frac{d}{dr}(1-r) = (1-r)^{-2} = \left(\frac{1}{(1-r)^2}\right) = \frac{1}{(1-r)^2}$$

Integration Basics:

• **Substitution:**

$$\int 2x \cos x^2 \, dx = ?$$

$$= \int \cos z \, dz$$

$$= \sin z + c$$

$$= \sin(x^2) + c$$

$x^2 = z$

$\Rightarrow 2x = \frac{dz}{dx}$

$\Rightarrow 2x\,dx = dz$

In calculus, integration by substitution, also known as u-substitution, reverse chain rule or change of variables is a method for evaluating integrals and antiderivatives. It is the counterpart to the chain rule for differentiation and can loosely be thought of as using the chain rule "backwards".

Method of Partial fraction:

The partial fraction is the result of writting a rational expression as the sum of two or more fractions. First simplify the rational expression by breaking it down into possible factors for the numerator, and denominator. Further, Split the expression into partial fractions based on the formulas. Example:

$$\left[\int \frac{A}{x+1} + \int \frac{B}{x-3} \right].$$

Sample Problem:

Evaluate $\dfrac{5x-3}{(x+1)(x-3)}$.

Solution:

$$\frac{5x-3}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$$

$\Rightarrow 5x-3 = A(x-3) + B(x+1)$

$\Rightarrow 15-3 = 0+4B \; ; x=3$

$\therefore B = 3$

$x = -1$

$5(-1) - 3 = A(-1-3) + B \times 0$

$\Rightarrow -5-3 = -4A$

$\therefore A = 2.$

--- X ---

Lecture-3

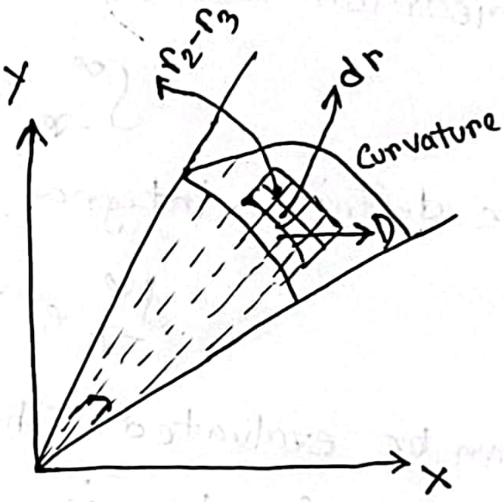
Intregration and Differentiation:

- Substitution
- Integration
- Double integral

$$\iint_D f(x,y) \frac{dxdy}{dA}$$

// Cartesian plane (x,y plane)
↳ Polar coordinate (r,θ)

$\iint e^{-(x^2+y^2)} dxdy = ?$



Let,

$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 = r^2(\sin^2\theta + \cos^2\theta)$

$= r^2$

$dxdy = rdrd\theta$

$$dxdy = \int_0^{2\pi}\int_0^{\infty} e^{-r^2} r\,dr\,d\theta = \int_0^{2\pi}\left(\frac{1}{2}\int_0^{\infty} e^{-u} du\right)d\theta$$

$$\left[\begin{array}{l} r^2 = u \\ \Rightarrow 2rdr = du \end{array}\right.$$

$$= \int_0^{2\pi} \frac{1}{2}\left[e^{-\infty} - e^0\right]$$

$$= \frac{1}{2}\int_0^{2\pi} 1 \cdot d\theta$$

$$= \frac{1}{2}[\theta]_0^{2\pi}$$

$$= \frac{2\pi}{2}$$

$$= \pi$$

কোণের বৃত্তীয় মান

$\Delta\theta = \frac{\text{চাপ}}{\text{ব্যাসার্ধ}}$

$\Delta\theta = \frac{P}{r_1}$

$\therefore P = r_1\Delta\theta$

$D = r_1\Delta\theta \cdot \Delta r$

$\parallel\!\!\mid$

$r_1 \approx r \approx r$

↳ $rdrd\theta$

** Gaussian

This integral from statistics and physics is not to be confused with Gaussian quadrature, a method of numerical integration. The Gaussian integral, also known as the Euler-Poisson integral, is the integral of the Gaussian function $f(x) = e^{-x^2}$ over the entire real line. Named after the German mathematician Carl Friedrich Gauss, the integral is.

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

The definite integral,

$$\int_{-\infty}^{\infty} e^{-x^2} dx$$

can be evaluated. The definite integral of an arbitrary Gaussian function is.

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx = \sqrt{\frac{\pi}{a}}$$

also,

$$\int_{-\infty}^{\infty} e^{-x^2} dx$$

Let,

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx$$

$$\Rightarrow I \cdot I = \int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$\Rightarrow I^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx dy$$

$$\Rightarrow I^2 = r$$

$$\therefore I = r$$

$^nP_r \equiv n$ permute r $\equiv \dfrac{n!}{(n-r)!}$

$$^nC_r \equiv \frac{n!}{(n-r)!\,r!}$$

Suppose,

$r = 3$

$r! = 3!$

$= 3 \times 2 \times 1$

$= 6$

$$\begin{bmatrix} abc \\ acb \\ bca \\ bac \\ cab \\ cba \end{bmatrix} \rightarrow 1 \text{ entry for } {}^nC_r$$

$^nC_r = C(n,r)$

$= \binom{n}{r}$

also used as the coefficient of Binomial expression.

Binomial expression (द्विपदी विस्तृति) :

$$(x+y)^n = \binom{n}{0} x^{n-0} y^0$$

$$(x+y)^1 = x+y$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x+y)^3 = x^3y^0 + 3x^2y + 3xy^2 + y^3 \cdot x^0$$

Ex:

$$(x+y)^n = \binom{n}{0} x^{n-0} y^0 + \binom{n}{1} x^{n-1} y^1 + \cdots\cdots + \binom{n}{r} x^{n-r} y^r$$

$n=3$,

$$(x+y)^3 = \binom{3}{0} x^{3-0} y^0 + \binom{3}{1} x^{3-1} \cdot y^1 + \binom{3}{2} x^{3-2} \cdot y^2 + \binom{3}{3} x^{3-3} y^3$$

$$= \frac{3!}{(3-0)!\,0!} x^3 + \frac{3!}{(3-1)!\,1!} x^2 y + \frac{3!}{(3-2)!\,2!} x y^2 + \frac{3!}{(3-3)!\,3!} x^0 y^3$$

$$= x^3 + 3x^2 y + 3xy^2 + y^3$$

<u>Pascal's identity</u>: The previous equation $(x+y)^3$ can be summerise by pascal's identity.

$(x+y)^n$

$n=0$ ------ 1

$n=1$ ---- 1   1

$n=2$ ---- 1   2   1

$n=3$ ---- 1   3   3   1

$$\binom{n+1}{k=2} = \binom{n=2}{k=2} + \binom{n}{k-1}$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

** pascal triangle ⇒ is the triangular arrangment of numbers that gives the coefficient in the expansion of any binomial expression

** Taylor expansion

=o=