CSE422

Lecture 4 & 5 Notes

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For modelling random experiments using probability theory we need 3 things. They are:

1. Sample space
2. Event space, and
3. Probability measure.

Sample space: is the list of all possible outcomes of a random experiment. It is a set denoted by **S**. Each element of S represents an outcome of the experiment.

Experiment		Sample space
● Rolling a dice	:	{H,T}
● Rolling a dice	:	{1,2,3,4,5,6}

Event space: is a collection of subsets of sample space. It is denoted by **F(S)**. We know that the power set is the set of all possible subsets of a set. Therefore,

$$F(S) \subset P(S)$$

For a finite sample space, we consider the whole power set as the event space. Therefore, event space for rolling a dice once is { {H}, {T}, {H,T}, ϕ }. Here ϕ is an impossible event and {H,T} is the sample space itself.

Probability measure: is a function that assigns a probability (ranging from 0 to 1) to each event in the event space. The symbol **P(.)** is used to denote it.

$$P(.) :\ F(S) \rightarrow [0, 1]$$

Keep in mind that the probability of an impossible event is always zero. Or, P(ϕ) = 0.

Dealing with infinitely large event space: Sample space could be of different sizes, both finite and infinite. Dealing with finite sample space is easy. However, the difficulty arises when we have to deal with infinite sample space. If the cardinality (means the number of elements of a set) of sample space is infinite, then the cardinality of event space, F(S) = 2^{∞}, which is even larger. In previous lectures, we learned that some infinitely large sets are countable too. For those countably infinite sets, we use the **σ-field** to reduce the size of the event space.

Three closure properties are used to define the σ-field. Those are:

1. If event A ϵ F(S), then A^c ϵ F(S)

2. If A_i ϵ F(S) for i = 1, 2, 3, … n then

$$\bigcup_{i=1}^{n} = A_i \ \epsilon \ F(S)$$

$$= A_1 \cup A_2 \cup A_3 \ \cup \ \ldots \ \cup \ A_n \ \epsilon \ F(S)$$

3. If A_i ϵ F(S) for i = 1, 2, 3, … ∞ then

$$\bigcup_{i=1}^{\infty} = A_i \ \epsilon \ F(S)$$

$$= A_1 \cup A_2 \cup A_3 \ \cup \ \ldots \ \cup \ A_\infty \ \epsilon \ F(S)$$

A set F(S) that satisfies the 3 closure properties as stated above is known as **σ-field**.

N.B. According to closure property-1 we can say that

- If ϕ ϵ **F(S)**, then **S** ϵ **F(S)** [Because complement of null set is the sample set]

Axiom: is a basic statement that is accepted to be true without any proof or derivation. Four important axioms for probability measure are given below.

Axiom 1: P(A) ≥ 0, P(A) ≤ 1 ⇒ 0 ≤ P(A) ≤ 1

Axiom 2: P(S) = 1

Axiom 3: If A_1, A_2, A_3, … A_n are countable collections of finite events and they are disjoint or mutually exclusive, then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + P(A_3) + \ldots + P(A_n)$$

Axiom 4: If A_1, A_2, A_3, … A_∞ are countable collections of finite events and they are disjoint or mutually exclusive, then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + P(A_3) + \ldots + P(A_\infty)$$

Prove that P(ϕ)=0

If Sample space is S, then

Event space, F(S) = {S, ϕ}

Here,

S = S U Φ

Or, P(S) = P(S U Φ)

Or, P(S) = P(S) + P(Φ) [using Axiom 3]

Or, P(S) - P(S) = P(Φ)

∴ P(Φ) = 0

Prove that P(A^C) = 1-P(A)

Let, A ⊂ S

∴ A U A^C = S [using closure property 1]

⇒ P(A U A^C) = P(S)

⇒ P(A) + P(A^C) = P(S)

⇒ P(A) + P(A^C) = 1 [using Axiom 2]

⇒ P(A) = 1 - P(A^C)