CSE422

Lecture 8 & 9 Notes

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PDF stands for Probability Density Function. It characterises continuous random variables. It is helpful in calculating probabilities when the sample space is uncountable.

A valid PDF function must fulfil the 2 properties as given below.

1. $f(r) \geq 0$, where r belongs to set of real numbers
2. $\int_{-\infty}^{+\infty} f(r)\, dr = 1$

Conditional Probability

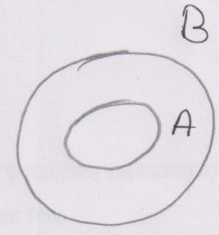
→ To update the prediction

→ Conditional probability has the ability to enhance or ~~update~~ improve to update of a probability

The conditional probability of an event A assuming that another event B has occured, denoted by $P(A/B)$

By definition,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \text{ where } P(B) \neq 0$$



if $A \subset B$ then $A \cap B = A$

$$\text{so, } \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \geq P(A)$$

if $B \subset A$ then $A \cap B = B$

$$\text{so, } \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

By definition,

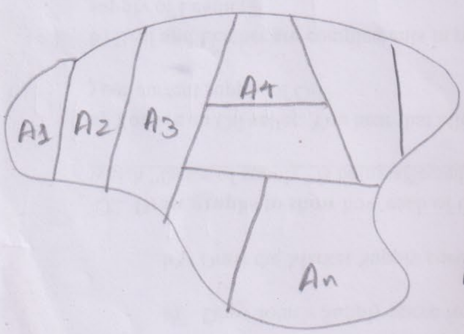
$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(B/A)\,P(A)$$

$$\Rightarrow P(A/B)\,P(B) = P(B/A)\,P(A)$$

$$\Rightarrow P(A/B) = \frac{P(B/A)\,P(A)}{P(B)}$$

S



Sample space

Here, $U = [A_1, A_2 \ldots A_n]$ is a partition of S

Which means

1. $A_1 \cup A_2 \cup A_3 \ldots . A_n = S$
2. $A_i \cap A_j = \emptyset \quad \forall i \in \mathbb{N}, \forall j \in \mathbb{N}$

So, the total probability law

$$P(B) = P(B/A_1)\,P(A_1) + P(B/A_2)\,P(A_2) + \ldots . + P(B/A_n)\,P(A_n)$$

where B is an arbitrary event

Proof: As S is a sample space &

B is an arbitrary event of that space

then we can say $B = B \cap S$ as $B \subset S$

As we all know an event is a subset of sample space.

So, $B \cap S = B \cap (A_1 \cup A_2 \cup \ldots A_n)$

$$= (B \cap A_1) \cup (B \cap A_2) \cup \ldots (B \cap A_n)$$

But the events $(B \cap A_1)$ & $(B \cap A_2)$ are mutually exclusive as the events A_1 & A_2 are mutually exclusive. Hence

$$P(B) = P(B \cap A_1) + P(B \cap A_2) + \ldots + P(B \cap A_n)$$

$$= P(B/A_1)P(A_1) + P(B/A_2)P(A_2) + \ldots + P(B/A_n)P(A_n)$$

So, Total probability law:

$$P(B) = \cancel{P(B \cap A} \; P(B/A_1)P(A_1) + P(B/A_2)P(A_2) + \ldots + P(B/A_n)P(A_n)$$

Now $S = \bigcup_{i=1}^{n} A_i$ $\quad P(B) = P(B \cap S)$

$$= P\left(B \cap \left(\bigcup_{i=1}^{n} A_i\right)\right)$$

$$= P\left(B \cap (A_1 \cup A_2 \cup \dots A_n)\right)$$

$$= P\left((B \cap A_1) \cup (B \cap A_2) \cup \dots (B \cap A_n)\right)$$

$$= P\left((B \cap A_1) + (B \cap A_2) + \dots + (B \cap A_n)\right)$$

$$= \sum_{i=1}^{n} P(B \cap A_i)$$

$$= \sum_{i=1}^{n} P(B/A_i)\,P(A_i)$$

$$P(B/A_i) = \frac{P(B \cap A_i)}{P(A_i)}$$

$$\Rightarrow P(B \cap A_i) = P(B/A_i)\,P(A_i)$$

$$P(A_i/B) = \frac{P(B \cap A_i)}{P(B)}$$

$$= \frac{P(B/A_i)\,P(A_i)}{\sum_{i=1}^{n} P(B/A_i)\,P(A_i)}$$

~~Practi~~ Practice Problem : 01 →

Tournament : 3 types of Players

A → ~~A~~ 50% are type A

B → 25% players are in this category

C → 25% " " " " "

My winning chance against A, B & C are 0.3, 0.4 & 0.5 repectively

What is ~~the~~ ~~pass~~ probability you win the match?

$P(W) = ?$

$$P(W) = P(W/A)\,P(A) + P(W/B)\,P(B) + P(W/C)\,P(C)$$

$$= (0.3 \times 0.5) + (0.4 \times 0.25) + (0.5 \times 0.25)$$

$$= \cancel{0.15 + 0.65 + 0.125}$$

$$= \cancel{0.925} = 0.15 + 0.10 + 0.125$$

$$= 0.375$$

So, I have 37% chance to win the game.

(Ans)

What is the probability you played against group A, given that you won.

$$P(A/W) = \frac{P(A \cap W)}{P(W)} \qquad P(A \cap W) = P(W/A)\,P(A)$$

$$= \frac{0.3 \times 0.5}{0.375}$$

$$= 0.40 \quad \text{(Ans)}.$$