Q1: Show that the Gaussian pdf

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ \frac{-(x-\mu)^2}{2\sigma^2} \right\}$$

satisfies the condition

$$I = \int_{-\infty}^{\infty} f_X(x)\, dx = 1.$$

Hint: It might be easier to find I^2 and then determine I.

Q2: Let X have exponential distribution

$$f_X(x) = \frac{1}{\mu} e^{-x/\mu} 1_{[0,\infty)}(x).$$

Find the conditional density $f_X(x|\mu < X \leq 2\mu)$.

Q3: (*Papoulis* 4-19) Show that

$$F_{\mathbf{X}}(x|A) = \frac{P(A|\{\mathbf{X} \leq x\})F_{\mathbf{X}}(x)}{P(A)}.$$

Q4: (*Papoulis* 4-21) The probability of *heads* of a random coin is a random variable $\mathbf{p}$ uniformly distruted on the unit interval $(0,1)$. **(a)** Find $P(\{0.3 \leq \mathbf{p} \leq 0.7\})$.**(b)** The coin is tossed 10 times and *heads* shows 6 times. Find the *a posteriori* probsbility that $\mathbf{p}$ os between 0.3 and 0.7.

Q5: (*Papoulis* 5-2) Find $F_{\mathbf{Y}}(y)$ and $f_{\mathbf{Y}}(y)$ if $\mathbf{Y} = -4\mathbf{X} + 3$ and $\mathbf{X}$ is an exponentially distributed random variable with p.d.f. $F_{\mathbf{X}}(x) = 2e^{-2x} \cdot 1_{[0,\infty)}(x)$.

Q6: (*Papoulis* 5-4) If $\mathbf{X}$ is a uniformly distributed random variable on the interval $(-2c, 2c)$, where $c > 0$, and $\mathbf{Y} = \mathbf{X}^2$, find and sketch $f_{\mathbf{Y}}(y)$ and $F_{\mathbf{Y}}(y)$.

Q7: (*Papoulis* 5-7(a)) We place 200 points at random in the interval $(0, 100)$. The distance from 0 to the smallest of the 100 points is the random variable $\mathbf{Z}$. Find $F_{\mathbf{Z}}(z)$.

Q8: (*Papoulis* 5-9) Express the density $f_{\mathbf{Y}}(y)$ of the random variable $\mathbf{Y} = g(\mathbf{X})$ in terms of $f_{\mathbf{X}}(x)$ if $\mathbf{Y} = g(\mathbf{X})$ when (a) $g(x) = |x|$, and (b) $g(x) = e^{-x} \cdot 1_{[0,\infty)}(x)$.